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Stress-Strain Curves of Fibrous Concrete in Compression

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Abstract

A simple experimental technique was used to obtain the stress-strain curves of the normal weight concrete containing steel fibers.

An analytic expression for the stress-strain relationship of the normal weight fibrous concrete is developed. Equations to predict the ultimate strain and the block stress parameters of the fibrous concrete were developed to reflect experimental results. The analytic expression has four constants, which depend on the properties of both the ascending and descending portion of the stress-strain curve and can be evaluated from the knowledge of the four key points of the curve.

Keywords:- Concrete, Compression, Fiber, Normal Weight, Stress, Strain, …etc.

Introduction

Reinforced concrete structures under load depend to a large degree on the stress strain relationship of concrete in compression (since concrete is used mostly in compression), such a curve is obtained by appropriate strain measurements in cylinder tests. Stress-strain curves consist of an initial relatively straight elastic portion in which stress and strain are closely proportional, then begin to curve to the horizontal, reaching the maximum stress, i.e., the compressive strength, at a strain that ranges from about 0.002 to 0.003 for normal density concrete[1]. Stress-strain curve of concrete in compression has a descending branch after the peak stress is reached; the characteristics of the curves after peak stress are highly dependent upon the method of testing. If special procedures are followed in testing to insure a constant strain rate while cylinder resistance is decreasing, long stable descending branches can be obtained [2].

Fiber reinforced concrete is an ordinary concrete with randomly dispersed discrete fibers, different types of fibers have been used to strengthen the concrete and the cement mortars in the field, these types are: asbestos, steel, carbon, glass, polypropylene and polyethylene fibers [3]. The most common type of fibers used with concrete materials is the steel fiber, which has a better compatibility with cement, based matrices [4]. It has been shown by many researchers [3,4,5] that the addition of steel fibers improve concrete properties in compression, tension, shear,

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**Part of his Ph.D.Thesis.*

flexure, ductility, impact, resistance, …etc.

Many investigators have tried to represent the stress-strain relationship of concrete by standard mathematical curves, e.g., a parabola, hyperbola, ellipse, cubic parabola or combinations like a parabola with a straight line or a sine wave with a cubic parabola and so on [6]. Because of the former curves are symmetrical about an axis, they can represent only the ascending position of the stress-strain curve of the concrete, hence, the necessity for defining the descending portion of the curve appears to have been the cause of the development of the various combinations [7].

Prakash and krishan[6] proposed an equation to represent the stress-strain relationship of concrete for short term loading as follows:

$$f = \frac{E \in}{1+(\in / \in_o)^2} \quad ----- 1$$

where:

E: modulus of elasticity of concrete

$\in$: Strain of concrete

$\in_o$: Strain of concrete at peak stress $\in_o = 0.00177$

They found that the area under stress-strain curve equal to 0.8 f`c $\in$o and depth of neutral axis equal to 0.692d. Chein and Carson[8] studied the effect of length, volume fraction and curing age on the direct compression strength of the concrete reinforced with random wires. Results had shown that the larger values of the modulus of elasticity, ultimate load and ultimate strain were obtained when steel fiber added to the concrete matrix. Hughes and Fattuhi [9] got the maximum increase in compression about 7% by using deform steel fiber (0.25x25mm). The change in the slope of stress-strain curve of the tested concrete cubes occurred between (65-75) % of the ultimate strength for the descending part. Wang et al[2] derived analytically the following equation to describe the stress-strain relationship (Fig.1) of concrete in compression.

$$y = \frac{Ax + Bx^2}{1 + Cx + Dx^2} \quad ----- 2$$

where:

y = f / fc`

x = $\in / \in_0$

f, $\in$: stress and strain in general.

fc`, $\in_0$: peak stress and corresponding strain.

A, B, C and D: constants.

Eo = fc` / $\in_0$: secant modulus of elasticity at peak stress.

Ec : secant modulus of elasticity at 0.45 fc`.

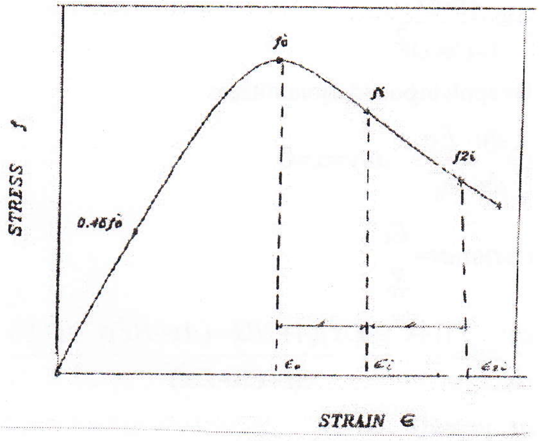


Fig. 1 Stress- strain relationship proposed by wang et al (Ref-2)

The accuracy of the above expression (Eq.2) in representing the stress-strain curve much improved when two separate sets of values of the four constants A,B,C and D were used for the ascending and descending parts. For the ascending part the values of the four constants were evaluated from the following four conditions:

1. $\frac{dy}{dx}=\frac{Ec}{Eo}$ for $y = 0,\ x = 0$
2. $y = 0.45$ for $x = 0.45/(Ec/Eo)$
3. $y = 1$ for $x = 1$
4. $\frac{dy}{dx} = 0$ for $y = x = 1$

For the descending part the condition for determining the four constants were:

1. $y = 1$ for $x = 1$
2. $\frac{dy}{dx} = 0$ for $y = x = 1$
3. $y = \frac{f_{1i}}{fc`}$ for $y = \frac{\in_{1i}}{\in_o}$
4. $y = \frac{f_2 i}{fc`}$ for $x = \frac{\in_2 i}{\in_o}$

Where:

f_{1i} and $\in_{1i}$: stress and strain at the inflection point.

f_{2i} and $\in_{2i}$: refer to a point which was arbitrary selected, such that, $\in_{2i} - \in_{1i} = \in_{1i} - \in_o$

Experimental work

Concrete with mix proportion of (1:2:2), (cement; sand; gravel) with water cement ratio of 0.57 have been used. Crushed aggregate with maximum size of 9.5 mm, river sand within ASTM[10] specification, hooked steel fiber with 0.5mm diameter and 50mm in length with different volume fraction (0.5, 1 and 1.5)% were used.

Mixing of materials done in a rotary mixer with a capacity of $0.1m^3$. Materials (sand and gravel) mixed dry, 50% of the water added, then the cement and remained water added, after the mixture become uniform, the steel fibers added slowly to prevent bundling of the fibers and to insure random distribution. The mixture molded into standard (150× 300) mm cylinders of steel fiber, after 24 hours the molds were stripped and kept moist for a period of 26 days, testing has been achieved at 28 days.

Stress-strain curves for the specimens has been plotted by using gauge type (Amil CCM20) as shown in Fig.2, which has been mounted to Avery – Denison testing machine with a maximum capacity of 100 ton.

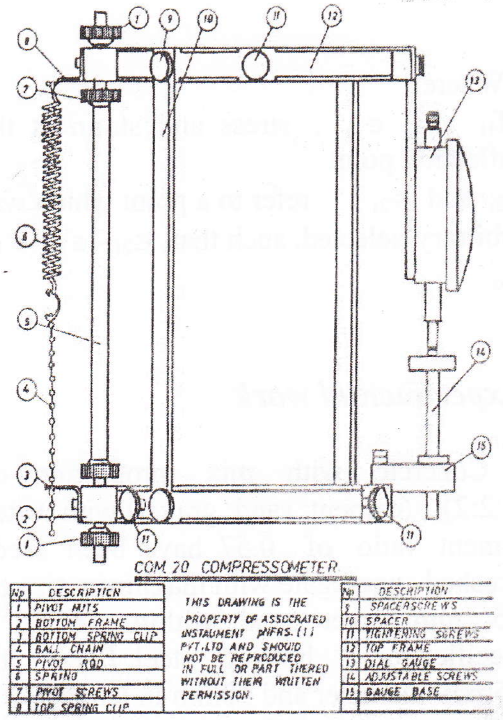


Fig.2 compressometer

Proposed Equations

It had been shown that the proposed equation by Wang et al [2] was the most acceptable relation for stress strain curve for plain and normal weight concrete, the relationship depends on the ultimate stress (fc`) and the ultimate strain ($\in_u$). The ultimate stress (fc`) was obtained from the experimental work while the ultimate strain ($\in_u$) obtained from Eq.(20) (it will be proposed later).

The stress – strain determined analytically was compared with data obtained from the experimental work.

Evaluation of Constants for Ascending Part:

$$y=\frac{Ax+Bx^2}{1+Cx+Dx^2}$$

By applying boundary condition one

$$(1)\ \frac{dy}{dx}=\frac{Ec}{E_0} \quad \text{at } y=x=0$$

$$\text{assume } n=\frac{Ec}{E_0}$$

$$\frac{dy}{dx}=n=\frac{(1+Cx+Dx^2)(A+2Bx)-(Ax+Bx^2)(C+2Dx)}{(1+Cx+Dx^2)^2}$$

$$at\ \ y=x=0$$

$$n=\frac{(1+0+0)(A+0)-(0+0)(C+0)}{(1+0+0)^2}$$

$$n=A$$

From boundary condition two

(2) $y=0.45$ for $x=0.45/n$

$$0.45+0.45\frac{0.45}{n}C+0.45\frac{0.45^2}{n^2}D=A(0.45/n)+B(0.45/n)2$$

$$C+\frac{0.45}{n}D=\frac{1}{n}B \qquad \ldots(4)$$

Boundary condition three

(3) $y=1$ at $x=1$

$$1=\frac{A+B}{1+C+D}$$

$$A+B-C-D=1 \qquad \ldots(5)$$

Finally by applying boundary condition four

(4) $dy/dx=0$ at $y=x=1$

$$(1+Cx+Dx^2)(A+2Bx)-(Ax+Bx^2)(C+2Dx)=0$$

at $x=1$

$$B=(Dn-n)/(C+2) \qquad \ldots(6)$$

Four equations and four unknowns A, B, C, D

from Eq.3, $A = n$

from Eq4, $B = nC + 0.45D$ ...(7)

from Eq.5, $B = 1 + C + D - n$ ...(8)

by equating equation (7) and equation (8), get

$$nC + 0.45D = 1 + C + D - n$$

$$nC - 1 - C + n = D - 0.45D$$

$$0.55D = (n-1)(C+1)$$

$$D = \frac{1}{0.55}(n-1)(C+1) \qquad ...(9)$$

By equating equation (6) and equation (7), get

$$\frac{Dn-n}{C+2} = nC + 0.45D$$

$$Dn - n = nC^2 + 0.45CD + 2nC + 0.9D$$

$$D = (nC^2 + nC + 2nC)/(n - 0.45C - 0.9) \qquad .....(10)$$

By equating equation (9) and equation (10) get

$$\frac{1}{0.55}(n-1)(C+1) = \frac{(nC^2 + n + 2nC)}{(n - 0.9 - 0.45C)}$$

$$K_o = \frac{0.55}{n-1}$$

$$C + 1 = K_o \frac{nC^2 + n + 2nC}{n - 0.9 - 0.45C}$$

$$nC - 0.9C - 0.45C^2 + n - 0.9 - 0.45C = nC^2K_o + nK_o + 2nK_oC$$

$$(0.45 + nK_o)C^2 + (2nK_o - n + 0.9 + 0.45)C + nK_o - n + 0.9 = 0$$

$$C^2 + \frac{(2nK_o - n + 1.35)}{0.45 + nK_o}C + \frac{nK_o - n + 0.9}{0.45 + nK_o} = 0$$

$$C^2 + K_1C + K_2 = 0$$

where

$$K_1 = \frac{2K_on - n + 1.35}{K_on + 0.45}$$

$$K_2 = \frac{nK_o - n + 0.9}{nK_o + 0.45}$$

$$C = \frac{-K_1 \pm \sqrt{K_1^2 - 4K_2}}{2} \qquad ...(11)$$

Evaluation of Constants For Descending Part

By applied conditions for the descending part, get:

(1) At peak point $y = 1$ for $x = 1$

$$1 = \frac{A+B}{1+C+D}$$

$$A + B = 1 + C + D$$

$$A = 1 + C + D - B \qquad ...(12)$$

(2) At peak point $dy/dx = 0$ for $y = x = 1$

In the present investigation a general solution for eight constants is made and these constants are determined by using computer program.. The values of the Eo, Ec, fc`, $\in_u$ and the eight constants for ascending and descending parts of the tested cylinders for each specimen are given in table (1).

From the analytic and experimental data, the following equations can be predicted to calculate the ultimate strain of concrete and block stress parameters for different values of volume fraction of steel fiber.

$\in_u = 0.003 + 0.008\,(V_f L/D)$

$\gamma_f = 0.85 + 0.02\,(V_f L/D)$

$\beta_f = 0.85 + 0.03\,(V_f L/D)$ for fc`<27.58 MPa.

$$\beta_f = 0.85 + 0.03V_fL/D - (1 + 0.25V_fL/D)(0.05\,\frac{fc' - 27.58}{6.86}$$

for 27.58 < f`c < 55.2 MPa.

$\beta_f = 0.65$ for fc` > 55.2 MPa.

As shown, the experimental curves compared with an analytic curve, from this comparison it appears that the analytic expression is quite satisfactory.

Table (1) Values of constants a. properties

Specimen No.	Vf %	$\in_o$ 10^{-3} mm/mm	fc` MPa	Ec kN/mm^2	Eo kN/mm^2
Group 1	0.00	3.00	17.32	21.21	5.77
Group2	0.50	3.37	17.77	21.71	5.26
Group3	1.00	3.75	18.20	22.81	4.86
Group4	1.50	4.00	18.82	23.81	4.70

b. constants

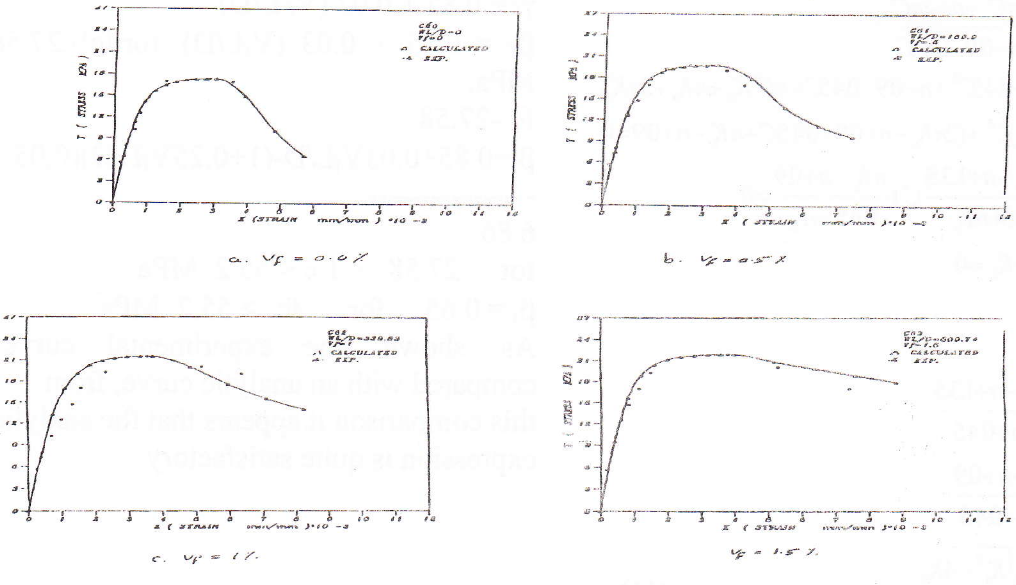
Specimen No.	**Ascending**				**Descending**			
	A_1	B_1	C_1	D_1	A_2	B_2	C_2	D_2
Group 1	3.67	12.00	1.67	13	2.27	-0.85	-0.37	0.79
Group2	4.13	16.78	2.13	17.78	1.54	-0.18	-0.95	1.13
Group3	4.71	24.03	2.71	25.03	1.29	0.36	-1.10	1.71
Group4	4.95	27.41	2.95	28.41	4.13	0.32	1.71	1.74

Typical idealized stress-strain relationship for plain and fiber concrete cylinders are shown in Fig. (3).

Conclusions

The stress-strain relationship of fibrous concrete was proposed based on the data from experimented work. Eight constants (A_1, B_1, C_1, D_1, A_2, B_2, C_2 and D_2) calculated by using computer program to define stress-strain relationship.

The ultimate strain of concrete in compression zone is increased with addition of steel fibers. The maximum increase is 33% observed by using volume fraction of 1.5%, the block stress parameters (γ_f, β_f) also increased by addition of steel fibers, finally the equations were proposed to predict of $\in_u$, γ_f and β_f.



a. Vf = 0.0 %

b. Vf = 0.5 %

c. Vf = 1 %

Vf = 1.5 %

Fig. 3 predicted and experimental stress-strain relationship for different volume fraction of fibers

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پەنوەندی نێوان فشارو جێ فشار لە کۆنکریتی تێکەڵاو بە ڕیشاڵی بۆڵایین لە ژێر بەستان

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پوختە

لەم لێکۆڵینەوەیەدا تەکنیکی تاقیگەی ئاسان بەکار هات بۆ دەربڕینی پەیوەندی لە نێوان فشار و جێ فشار لە کۆنکریتی ئاسایی تێکەڵاو بەریشاڵی بۆڵایین لەژێر پەستان.

توانرا گەشە بدرێت بە پەیوەندی نێوان فشار و جێ فشار بەشێوەی تیۆری بۆ کۆنکریتی ئاسایی، گەشەدان بەهاوکێشەکان بۆ پێشبینی جێ فشاری کۆنکریت وەگەشەدان بەهاوکۆلکەی فشار بۆ بڕگەی هاوسەنگ بۆ کۆنکریتی تێکەڵاو بەریشاڵی بۆڵایین بۆ ڕەنگ دانەوەی ئەنجامی تاقیگەریەکان.

پەیوەندی تیۆری فشار و جێ فشار چوار نەگۆری هەیە، ئەو نەگۆڕانە پشت دەبەستن بەرەوشتی بەشی سەرکەوتو ascending وبەشی داکەوتوو descending لە پەیوەندی فشار و جێ فشار، وەدەتوانرێت ئەم نەگۆڕانە حیساب بکرێت بەگوێرەی چوار خاڵ لەسەر پەیوەندیەکە.

منحنيات الإجهاد - الانفعال لخرسانة الحاوية على الألياف الفولاذية تحت الانضغاط

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الخلاصة

تم استخدام تقنية مختبرية بسيطة لإيجاد علاقة إجهاد – انفعال للخرسانة الاعتيادية والحاوية على الألياف الفولاذية تحت الانضغاط تم تطوير علاقة الإجهاد – الانفعال نظرياً للخرسانة المعززة بالألياف الفولاذية، تطوير معادلات للتنبؤ بأقصى انفعال للكونكريت (ultimate strain) وتطوير معاملات إجهاد للمقطع المكافئ (block stress parameters) للخرسانة المعززة بالألياف الفولاذية لينعكس النتائج العملية.

إن العلاقات النظرية للإجهاد – الانفعال لها ثوابت. هذه الثوابت تعتمد على خواص كل من الجزء الصاعد ascending والجزء النازل descending لمنحني الإجهاد – الانفعال ويمكن حسابها من خلال أربع نقاط على المنحنى.

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وەرگیرا لە٢٠٠٠/٤/٤دا پەسەندکرا لە ٢٠٠١/٥/١٧دا.